

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

- Obtain the power series solution of Legendre differential equation
  - Show that  $\frac{1}{2^n n!} \frac{d^n P_n}{dz^n}$  is the coefficient of  $z^n$  in the expansion of  $(-z + \sqrt{1-z^2})^{-1/2}$  in ascending powers of  $z$ .
- Find the solution of Bessel differential equations  $(z^2 \frac{d^2}{dz^2} + z \frac{d}{dz} + (z^2 - \nu^2))y = 0$
  - Discuss the recursion relations relating to Legendre Polynomial.
- State and prove Cauchy integral theorem

$$f(z) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x) dx}{z - x}$$

- Show that  $\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2} = \frac{\pi}{a}$

- State the Taylor's and Laurent's theorem.
  - Define poles and residues. Find out the poles and residues for the function

$$f(z) = \frac{1}{z^2 + 1}$$

- Explain the gradient and divergence of a tensor.
  - What is a Cartesian tensor? Classify the Cartesian tensors. Give the concept of derivative of a tensor.
- Discuss Riemann space associated tensors.
  - Explain Christoffel's symbols and transformation laws of Christoffel's symbols.
- Define a Laplace transform. State and explain first and second shifting theorems.
  - Define a Fourier transform. Obtain the Fourier transform of Delta function.
- Find the Laplace transform of the function  $F(t) = t \sin at$ .

- Find the Fourier transform of  $f(x) = \begin{cases} -|x| < 0 \\ |x| > 0 \end{cases}$  and hence evaluate  $\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2}$

- Answer any TWO of the following
  - Cauchy - Riemann equations and their applications.
  - Quotient law of tensors and difference between symmetric and antisymmetric tensors.
  - Laplace transform of a derivative and integral form.
  - Find the values of the Bessel function  $J_{\nu}(x)$  at  $x=0$ .